Discussion on the *Derivations* of *Newton's Laws*, *Law of Gravity*, and the *Gravitational Constant* as founded on *Fundamental Philosophical Principles* and *Formulated* using *Geometric* and *Numeric Reasonings* ©

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Abstract

Strobel's derivations of *Newton's Laws*, the *Law of Gravity*, and the *Gravitational Constant* (G) are based on *Most Fundamental Defacto–Apriori Philosophical Principles* and *Higher Level Ipso–Facto Principles* supporting *Geometric* and *Numeric* operations on *Standardized Metrics* of *Differential Orders* of *Generalized Momentums* and *Generalized Positions*. *Metrics* are defined in *Context* with *Normalized Magnitudes* in all *Differential Orders* and follow *Quantum Mechanics* type *Constructs* being *Forward* and *Inverse Operations* between two *Standardized Metric Spaces* where one is *Calibrated* and the other *Normalized*. The calibration is from the *Mathematical Construction* into the *Physical* uses the *Universal Calibration Constant* for *Mass* ($\mathfrak{m}$) determined from *Planck's Constant* and a reduced form for *Einsteins Mass—Energy Equation*. The terms of the general form approach *Zero* at appropriately low *Order* for *Newton Mechanics* and they are valid for all *Scientific Investigations* having *Primary Constants* of *Mass*, *Distance*, and *Time* calculated using *Standardized Units*.

The *Gravitational Constant* G is a *Context Dependent Universal Constant* that is shown not to be *Universal* as is the *Convention*. The derived *Gravitational Constant* is a prediction of the *Emperical Value* subject to the *Constraints* of the *Context*. Introducing *Constraints*—namely those of the *Center of Mass Context*—makes it a *Universal Constant* for *Standardized Derivations* in all *Contexts* subject to those *Constraints*. In *Classical Mechanics* they can be determined for any *Calibrated System* of *Measure* and converted to any other *System of Measurement* using only the conversion factor for the *Units of Measure* for *Mass*.

A value for G in *SI* units for *Mass/Time* ($gm/\sec$), is calculated with *Planck's Constant* implicit in it's *Emperical Value* from the more *Fundamental Universal Constant* for *Mass* $\mathfrak{m}$. A second, independent calculation for G is obtained using empirical *FPS* values for h and c and the conversion factor for *Mass* between *pounds* and *grams*. The variance between these two completely independent calculations for G^d from independent measurements for h is roughly of the *Order* of .22 parts in $1,000,000$. The *Emperical Value* for G^c must consider *Context* before accurately comparing to G^d. Two recent and independent measurements of G^c are .45 parts in $1,000$ different from each other while being .61 parts in $1,000$ and .65 parts in $1,000$ different from the calculated value for G^d. Considering *Context* gives an estimate for G improved by five orders of magnitude.

G is a *Transcendental Number* and *Planck's Constant* is the computed value in the *Inverse Operation* and is *Transcendental* and a *Universal Constant* based on it's *Conventional* treatment. The *Mathematical Equivalent* (N) for G is a *Most Fundamental Construct* of *Mathematics* that *Transforms* between two *Normalized Metric Spaces* as contrasted to G which *Transforms* between a *Normalized Metric Space* and a *Calibrated Metric Space*. The *Mathematics* of N falls in the field of *Modular Forms*.

Assuming bodies with *Homogeneous Density Mass Distribution* and in *Context*, the *Bulk Masses* for *Planets Modelled* in this simplest of cases are calculated to be half that accepted under conventional analysis.

Expanded Abstract

Discussed here are Strobel's derived expressions for *Newton's Laws*, the *Law of Gravity*, and the *Gravitational Constant* (G^c) as based upon *Most Fundamental Defacto–Apriori Philosophical Principles* and *Higher Level Ipso–Facto Principles*—*Context* and the *Fundamental Equation* (*FEq*)—supporting *Geometric* and *Numeric* operations on *Standardized Metrics* of *Differential Orders* of *Generalized Momentums* and *Generalized Positions*. Derivations start with *Geometric* constructs from the *Euclidean Distance* as the *Space Metrics* with *Numeric Methods* applied to *Infinite Series* and their *Partial Sums* evaluated in their *Infinite Limits*.

These *Metrics* are defined in the *Newtonian Mechanics Context* with *Normalized Magnitudes* in all *Differential Orders* and follow *Quantum Mechanics* type *Constructs* being *Forward* and *Inverse Operations* between two particular *Standardized Metric Spaces*. One is *Calibrated* and the other *Normalized*. The calibration is from the *Mathematical Construction* into the *Physical* realm uses the *Universal Calibration Constant* for *Mass* ($\mathbb{m}$) determined from *Planck's Constant* and a reduced form for *Einsteins Mass—Energy Equation*. The *Context* of these derivations is the *Center of Mass Context* which is in many ways analogous to the *Center of Mass Reference Frame* from *Classical Mechanics*. The *Space Metrics* are based on the "*Distance from Origin*" *Standardized Metric* as an *L–Function Equivalent* to a *Modular Form*. They are calibrated to the *Physical Domain* using the *Universal Calibration Constant* for *Mass* $\mathbb{m}$. The terms of the general form approach *Zero* at appropriately low *Order* in the *Newtonian Mechanics Context*. These are generalized expressions for all *Contexts* having *Primary Constants* of *Mass*, *Distance*, and *Time* calculated using *Standardized Units* and with *Newtonian Mechanics* being one particular application.

Two G's are considered—the *Conventional Emperically Determined* (G^c) and that *Derived* by Strobel (G^d). G^c in this discussion becomes a *Context Dependent Universal Constant* and not generally *Universal*—as believed in *Conventional* treatments. Introducing *Constraints*—namely those of the *Center of Mass Context*—makes it a *Universal Constant* for *Standardized Derivations* in all *Contexts* subject to those *Constraints*. G^d is a *Universal Constant* that can be calculated for any *Calibrated System* of *Measure* and converted to any other *System of Measurement* using only the conversion factor for the *Units of Measure* for *Mass*. For example: $1.000...lb = 0.453592...gm$ converts between FPS *Units* and SI *Units*. G^d is the prediction of the *Emperical Value* for G^c subject to the *Constraints* of the *Center of Mass Context*.

A value for G^d in SI units for *Mass*/*Time* (gm/sec), is calculated with *Planck's Constant* implicit in it's *Emperical Value* from the more *Fundamental Universal Constant* for *Mass* $\mathbb{m}$. A second, independent calculation for G^d is obtained using empirical FPS values for h and c and the conversion factor for *Mass* between *pounds* and *grams*. The variance between these two completely independent calculations for G^d from independent measurements for h is roughly of the *Order* of .22 parts in $1,000,000$. The *Emperical Value* for G^c must consider *Context* before accurately comparing to G^d. Two recent and independent measurements of G^c are .45 parts in $1,000$ different from each other while being .61 parts in $1,000$ and .65 parts in $1,000$ different from the calculated value for G^d. Considering *Context* gives a derived value for G^c improved by four orders of magnitude.

G^d is a *Transcendental Number*. *Planck's Constant* is the computed value in the *Inverse Operation* and is *Transcendental* and a *Universal Constant* based on it's *Conventional* treatment in *Center of Mass Contexts*. The *Mathematical Equivalent* for G^d is a *Most Fundamental Construct* (N) that *Transforms* between two *Normalized Metric Spaces* as contrasted to G which *Transforms* between a *Normalized Metric Space* and a *Calibrated Metric Space*. Every *Physical Equivalent* G is directly related to this *Normalized Value* by some *Calibration Constant* for the *Principle Standard Metric* of the *Context*.

By reasonings of *Context* and assuming bodies with *Homogeneous Density Mass Distribution*, the *Bulk Masses* for *Planets Modelled* in this simplest of cases are calculated to be half that accepted under conventional analysis. This prediction is supported with comparisons against *Bulk Densities* calculated from prevalent densities of each *Planet* in the *Solar System*.

Introduction

This paper discusses a series of unpublished documents from Strobel [4, 5, 6, 9] in which *Newton's Laws*, the *Law of Gravity*, and the *Gravitational Constant* (G^d) are *Derived*. The discussion starts with *Most Fundamental Philosophical Principles* leading to the *Conventional Mathematical Representations* to which is referred to here as *Classical Mechanics*. The derivations are outlined in a number of *Mathematical Approaches* structured on *Geometric* and *Numeric* arguments. Shown are the *Law of Gravity* and *Newton's Laws* as *Forward* and *Inverse Transforms* for a *Standardized* "*Distance from the Origin*" *Metric*. The *Transform Calibrates Values* with a *Universal Physical Constant* for *Mass* (m) in the *Forward Direction* and the *Inverse Operation Normalizes* the *Calibrated Values* using it's *Multiplicative Inverse* ($\frac{1}{m}$). Universal *Geometric* and *Integer Constants* are involved and the result is a *Vector Expression* corresponding in part to the *Scalar* G^c of *Conventional Newtonian Mechanics*. The final steps in the formal derivations are purely *Mathematical* exercises relying on the *Calibration* from this one *Universal Physical Constant* to cast the *Normalized Mathematical Representation* into a *Calibrated Physical Model*.

A focus here is on the *Philosophical Basis* for the analysis but generalities of the *Mathematics* are also discussed. The *Philosophical Reasonings* establish that *Mathematical Developments* are possible and determine that the *Mathematical Relationships* between these *Physical Properties* are as stated, and are valid for all such *Scientific Studies*. Results of the derivations are included but the rigorous and complete *Philosophical* and *Mathematical* discussions are presented elsewhere.[*ibid*]

The *Mathematical Constructs* are the *Forward* and *Inverse Operations* between two *Metric Spaces*:

$\{M\} \mid M \equiv \{\{r_j\}_i \mid j \in [1,3]\&$

$$i \in [0,\infty]\&r_{s/j,i} \parallel r_{s/j,k} \mid r_{s/j,i} \perp r_{s/l,i} \forall l \neq j \forall k \in [0,\infty])\},$$

and:

$$\{m\} \mid m \equiv \{|\overset{i}{\vec{\rho}_s}| \mid |\overset{i}{\vec{\rho}_s}| \equiv m \cdot |\overset{i}{\vec{r}_s}| \in \mathcal{E}_\infty \mid |\overset{i}{\vec{r}_s}| \perp |\overset{j}{\vec{r}_s}| \forall j,i \in [0,\infty]\&i \neq j\},$$

based on the *Distance from the Origin Metric*. The result is a derived *L-Function Equivalent*:

$$|G| = \binom{4!}{2!} \cdot e^{\frac{1}{4}\left[\ln\left(\frac{m}{\varphi}\cdot\frac{1}{2\cdot e^4}\right)-1\right]},$$

given the *Modular Form* satifying the condition for *Analytic Continuation* and where ϕ is the *Golden Ratio*. This *Mathematical Construct* is calibrated to the *Physical Domain* using the *Universal Calibration Constant* for *Mass* (m) determined from a reduced form for *Einsteins Mass—Energy Equation*:

$$m \equiv h \cdot t_s \cdot \left(\frac{1}{c \cdot t_s}\right)^2 = 0.73724972014... \times 10^{-53} gm/sec.$$

The natural units for G^d are units of *Mass* divided by *Time* where the *Universal Constant* for *Time* is used as the "*Reference Standard Metric*". [1]

The inverse to the *L–Function* is:

$$|h| = c^2 \cdot \varphi \cdot e^{4\left[\ln\left(\frac{\frac{1}{|G|}}{\frac{4!}{2!}}\right)\right]+1},$$

where the natural units for h are units of *Mass* $\times$ *Distance* divided by *Time* and the *Universal Constant* for *Time* is used as the *Reference Standard Metric*. A derived value for this *Gravitational Constant* in *SI* units of *Mass*/*Time* ($gm/$sec) is calculated with *Planck's Constant* implicit by it's *Emperical Value* in the calculation for the *Universal Constant* m and results in a *Calculated Value* for G^d:

$$|\overset{0}{G}|^4 = |\overset{1}{G}|^4 = \left[\frac{1}{2} \cdot \left(\frac{4!}{2!}\right)^4 \cdot \frac{m}{\varphi \cdot e^5}\right]^4,$$

or equivalently:

$$= \left[\frac{1296}{(1+\sqrt{5}) \cdot e^5} \cdot m\right]^4 = (0.66785532506... \times 10^{-13} gm/sec)^4, \quad (1)$$

leading to the *Mathematical Identity*:

$$\frac{G \cdot c^2}{h} = \frac{1296}{(1+\sqrt{5}) \cdot e^5} \equiv N.$$

N is *Transcendental.*

The statement for the *Law of Gravity* in the *Newtonian Mechanics Context* is:

$$\{m \times \overset{l}{r_s} \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot M \cdot \left(\frac{m \cdot \overset{0}{G}}{{r_s}^l l!}\right) +$$

$$O^3\left(\frac{0.54784235257... \times 10^{-83}}{(0.299792458... \times 10^9)^l \cdot (l+2)!}\right) l \in [1, \infty],$$

while the *Generalized* statement for *Newton's Laws* are:

$$\frac{\overset{l}{\rho_s} \cdot l!}{\overset{0}{G}} = (-1)^{(l)} \cdot m \cdot \overset{l}{r_s} \overset{\mathcal{T}}{\leftrightarrow} 1 : \{l \in [0, \infty]\}.$$

For *Newtonian Mechanics Contexts*, *Terms* greater than the 2^{nd} *Differential Order* are not significant on the *Planetary* scale. This is not true for either the *Galactic* scale nor for the *Quantum* scale.

These are *Mathematical Treatments* of *Standardized Metrics* and involve the *Quantum Mechanical Representation* of *State Metrics* which are calibrated using the *Principle Calibration Constant* of *State—h.* h is a *Composite Universal Constant* composed of *Primary Calibration Constants* which are *Universal* i.e., they are *Context Independent* and thus the *Composite Constant* is also *Universal.*

The *Law of Gravity* has an analogical *Mathematical Identity* defined by the *Standardized Metric*, the *Distance from the Origin Metric*, which is defined by the *Euclidean Distance.* The *Euclidean Distance* is the *Space Metric* for these *Metric Spaces.* The *Mathematical Construct* arises from the *Standardized Metric*—the *Distance from the Origin Metric* in the *Ordered Normalized Infinite Dimensional Euclidean Space* as *Mapped* onto the *Distance from the Origin Metric* in a *Normalized Ordered Infinite Set of Three Dimensional Euclidean Spaces* with $m \equiv 1$:

$$|\overset{1}{N}|^4 = (1.2816774372....)^4,$$

where the $\overset{1}{N}$ is a scalar factor applied to a *Vector* having the form:

$$\vec{N} \equiv \{\overset{i}{N}\} \equiv \{\frac{\overset{0}{N}}{i!}\} \mid i \in [0, \infty].$$

$\vec{N}$ is the *Mathematical* analog to the *Standardized Physical Metric*—the *Physical Law of Gravity.*

In *Newtonian Mechanics*, but with the language of *Quantum Mechanics*, the representative state *Metrics* in the *Newtonian Mechanics* representation given in terms of the *Quantum Mechanics Representation* is:

$$\overset{l}{|1|}^4 = \left(\frac{h}{m\cdot c^2}\right)^4 = \left(\frac{\overset{l}{G}}{m\cdot \overset{l}{N}}\right)^4.$$

It has *Normalized Value* $(1)^4$ as defined by the *Roots of Unity* for the *Linear Form* with implicit statement of h within the *Constant* G^d. This explains why *Newton's Laws* and the *Law of Gravity* are complete representations for the *Laws of Physics* in the *Newtonian Mechanics Context* and why *Calibrated Universal Constants* like the *Speed of Light-c* and m can be determined in *Newtonian Studies* as readily as from *Quantum Mechanical Studies*. This is independent of any knowledge of a true *Mathematical Relationship* between h and G^c. The *Speed of Light-c Constant* is normally a *Quantum Mechanics Metric*. The *Newtonian Mechanics Context* deals fully with the usual physical events of our daily observations. It also shows there are *Quantum Mechanics Mathematical Representations* in *Newtonian Mechanics* and this is an unavoidable consequence from the two *Scientific Methodologies* having a common *Measurement Space*.

The *Domain* and *Range* of these *Normalized Maps* are the *Unit Spheres* about the *Origin* in the *Normalized Metric Spaces* and the *Mapping* is *One to One and Onto*. These *Unit Spheres* are *Generalized* N^{th}*-Dimensional Objects* defined by the *Differential Order* $k|k \in [0, \infty]$. *Complex Numbers* can be used in the analysis where *Ranges* of the *Unit Sphere* are considered off the *Real Valued Axis*.

Several of the strategies followed in the derivations start with *Geometric* arguments that depend explicitly on the *Euclidean Distance*—the *Space Metric* for the *Metric Spaces* involved. *Numeric Methods* with *Partial Sums* and their *Infinite Limits* are applied in all cases.

Two *Gravitational Constants* ($G's$) are essential to this discussion—the *Conventional Empirically* determined value G^c and the *Derived* value G^d. G^d is calculated using one *Universal Physical Constant* (m), one *Universal Mathematical Constant* (*Euler's Number*), and the two smallest *Prime Numbers* $(1, 2)$ defining 1 to be defined as a *Prime Number* and with $5 = 2^2 + 1^2$. G^d is a *Universal Constant* that can be calculated for any *Calibrated System* of *Measure* and converted using only the conversion factor for the *Units of Measure* for *Mass*; for example, $1.000...lb = 0.453592...gm$ in FPS verses SI *Units*. This is a *Mathematical Property* of *Modular Forms*. *Constraints* are introduced on *Elements States* that make it a *Universal Constant* for *Standardized Derivations* in all *Contexts* subject to those *Constraints*. Another property of G^d is that it is a *Transcendental Number*.

G^c is a *Context Dependent Universal Constant* and strictly not *Universal*. It is *Context Dependent* generally and *Context Independent* when *Constrained* and a *Composite* of *More Fundamental Primary Fundamental Constants* as from the *Universal Constant* for *Mass*. It is *Universal* to the *Quantum Mechanics Context*, but a truly *Universal Constant* is *Constant* in all places and at all times and for all *Contexts*, and thus would be *Context Independent*. The *Primary Calibration Constants* are *Context Independent* and therefore *Universal Constants* for all *Scientific Studies*.

Planck's Constant defined in a manner similar to G^d, is shown to be a *Transcendental Number* and a *Universal Constant*. h is *Context Independent* by *Constraint* and a *Composite* of *More Fundamental Primary Fundamental Constants* as from a reduced form of *Einsteins Mass—Energy Equation*. The role h plays in the *Quantum Mechanical Context* is in a sense the same as the role the *Speed of Light-c* plays when *Normalized* in the *Newtonian Mechanics Context*—they are both *Universal Calibration Constants* for *Principle Standard Metrics* in their respective *Contexts*. They are identified to be "*Principle Standard Metrics*".[1]

The three *Primary Calibration Constants* of *Newtonian Mechanics* and *Quantum Mechanics* are the *Speed of Light-c*, the *Universal Calibration Constant* for *Mass* (m) as presented here and from Strobel,[1] and the *Universal Calibration Constant* for *Time* ($t_s \equiv 1$).[ibid] The *Reference Standard Metric* of *Time* is arbitrarily given the *Calibration Constant* $1.000....sec$ with no loss of *Generality* as treated in *Conventional Analysis*. The *Universal Physical Constant* m is as critical to the *Scientific Understanding* of the *Physical Universe* as is the *Universal Physical Constant Speed of Light-c*. The *Universal Calibration Constant* for *Time* is equally important—more so as all other *Calibration Constants* are in *Reference* to it.

G^d plays a *Unique* role in *Scientific Studies* in that it *Transforms* between the *Normalized Metric Space* representing the *Newtonian Mechanics Context* and the *Calibrated Metric Space* representing the *Quantum Mechanics Context*. Both *Metric Spaces* are structured differently using different *Space Basis Metrics* and *Principle Standard Metrics*.

In the *Physical World*, the *Transform* is *Calibrated* in one direction using m and *Normalized* in the opposite direction using the *Multiplicative Inverse* of m. *Newton's Laws*, collectively and cast into an *Expanded Form*, are the *Inverse* to the *Law of Gravity* under this *Transformation* but in the opposite direction. The *Law of Gravity* must also be treated in *Expanded Form* in order to fully define the *Physical Properties* of a *Physical System*. Both statements of this same *Transform* with different directions are associated with the *Physical Properties* of *Planets*, *Stars*, and *Galaxies*, but as discussed here they apply to all *Physical Systems* including those of *Quantum Mechanics*.

These expressions are general for the *Standardized Units* as used for the measurements for *Mass*, *Distance*, and *Time*. A second independent computation of G is obtained using the FPS result for h and the conversion factor of *Mass*—*pounds* to *grams*. This calculation for G^c as obtained from independent FPS measurements for *Planck's Constant* in determining *Planck's Constant* in FPS units, where the conversion factor for *Mass* has been used with the accepted value for h, to cast the result from FPS into SI units. Greater consistency for the calculations using h is expected when compared with direct *Measurements* of G^d since the direct *Quantum Mechanics* measurements for h can be performed with greater precision than can be done by directly measuring G^c. Additional deviations will affect results when not considering the *Context* of *Measurements* which is a problem with *Conventional Studies* since *Convention* has no concept of *Context*. The corresponding calculated value of the *Universal Constant* G^d using *Emperical* h in FPS units:

$$h_{FPS} = 0.1068846... \times 10^{-8} lb/sec, \tag{2}$$

as *Transformed* into the SI units of measure using the conversion ratio of gm/lb is:

$$G' = 0.66785546992... \times 10^{-13} gm/sec. \tag{3}$$

The variance between these results—Equation (1) and Equation (3)—is roughly .22 parts in $10,000,000$. This preliminary finding is compelling support for the *Mathematical* and the *Philosophical Strategies* and for the predicted *Emperical Results* of Equations (1) and (3). However, the *Emperical Value* for G^c must consider *Context* before accurately comparing to G^d. Two recent and independent measurements of G^c are .45 parts in $1,000$ different from each other while being .61 parts in $1,000$ and .65 parts in $1,000$ different from the calculated value for G^d.[12] At first glance, considering *Context* produces an estimate for G improved by four orders of magnitude. Comparisons are tabled in Table (1).

Calibrated Standard Metrics which are the *Transformed Unit Values* from the *Newtonian Mechanics Context* to the *Quantum Mechanics Context*. All values are with respect to *Unit Time* (the *Reference Standard Metric*).

†—since the calibrated value for *Mass* is calculated from the measured value for *Planck's Constant*, the calculated value for the calibrated *Planck's Constant* will be the same as the measured. The *Normalized* value for *Momentum* is 1.
‡The *Newtonian Value* for G^d is taken to be the *Transformation* (N) between *Normalized Metric Spaces* to be consistent in principle with the treatment of *Normalized Values* as *Standardized Magnitudes* in the *Newtonian Context*.

Calculations involving different understandings of *Newton's Laws* and the *Law of Gravity* as with the conventional approach will yield different results when these differences are not accounted

G	description	Value of G	Δ (from)	Δ	Variance
G^d_{fps}	Derived fps	$.66785546992... \times 10^{-13}$	G^d_{SI}	$.14486 \times 10^{-19}$	$.21690 \times 10^{-6}$
G^d_{fps}	Derived fps	$.66785546992... \times 10^{-13}$	G^a	$.42547 \times 10^{-16}$	$.63727 \times 10^{-4}$
G^d_{fps}	Derived fps	$.66785546992... \times 10^{-13}$	G^c_1	$.14486 \times 10^{-19}$	$.21690 \times 10^{-6}$
G^d_{fps}	Derived fps	$.66785546992... \times 10^{-13}$	G^c_2	$.40707 \times 10^{-16}$	$.60970 \times 10^{-3}$
G^d_{SI}	Derived SI	$.66785532506... \times 10^{-13}$	G^a	$.42533 \times 10^{-16}$	$.63705 \times 10^{-3}$
G^d_{SI}	Derived SI	$.66785532506... \times 10^{-13}$	G^c_1	$.43693 \times 10^{-16}$	$.65444 \times 10^{-3}$
G^d_{SI}	Derived SI	$.66785532506... \times 10^{-13}$	G^c_2	$.40693 \times 10^{-16}$	$.60949 \times 10^{-3}$
G^c_1	Measured case 1	$.6674184 \times 10^{-13}$	G^a	$.11600 \times 10^{-17}$	$.17380 \times 10^{-4}$
G^c_1	Measured case 1	$.6674184 \times 10^{-13}$	G^c_2	$.30000 \times 10^{-17}$	$.44948 \times 10^{-4}$
G^c_2	Measured case 1	$.6674334 \times 10^{-13}$	G^a	$.18400 \times 10^{-17}$	$.27568 \times 10^{-4}$
G^a	Accepted	$.667430 \times 10^{-13}$			

Table 1: Variances for the values of G

This table contains the calculated variances between values of G^c and G^d. The derived values use the

for in the design of a *Scientific Study*. *Context* is key to the development here and states the *Measurement* and the *Modelling Strategies* of *Physical Properties* of *Physical Systems* determine the results for the *Scientific Study*.

Following the observation that the calculations for the *Masses* of the *Planets* involve two separate *Contexts* as in this *Philosophy*, and being this is not considered in *Conventional Results*, leads to the conclusion that currently accepted values for the *Masses* for the *Planets* are incorrect. The corrected values are tabled here and shown to be consistent with the assumption the *Planets* are more or less *Homogenious Bodies* interms of their *Mass Densities*. The *Physical Properties* of the *Planets* derived in terms of their *Bulk Densities* as calculated with most prevalent mass densities for each *Planet*. Most cases of the corrected results have *Mass Values* reduced by a factor of one-half when compared with measurements in the *Center of Sun Context*.

The *Values* are particularly close to the calculations for *Masses* for the *Inner Planets* using *Volume* x *Density Values* and this is expected since *Observations* are most accurate for those *Planets* nearest to *Earth*. It is conjectured here that these bodies comprise rocks in various states, but *Homogeneous* in-terms of their *Mass Density Distribution*, which is more likely than the *Outer Planets* distanced from the *Sun*'s influence and likely comprise uneven distributions of gasses, frozen gasses, and liquids as well as rock as dust and chunks of various sizes and densities based on origins. The calculated results are consistent between *Density—Volume* and corrected *Gravity Calculations* for *Mass* to an *Order* of less than $\approx 2\%$ for the *Inner Planets*. Results for *Mars* are significantly outside this range although still significantly better than the current accepted value for *Mass* and conjectures here are made in this regards.

The *Philosophical* Foundations

The Foundations of these developments are *Philosophical* following implicit principles identified to be *Most Fundamental* to all *Scientific Studies—Defacto-Apriori*. Reason leads from there to several *Direct Consequence Principles—Ipso Facto*.

The details start with an inventory of a set of the *Most Fundamental Principles*—approximately eighteen in number[3]—which are *Implicitly* accepted *Defacto-Apriori*. Those necessary to the understanding of the arguments here are paraphrased below:

- The *Three Component Model of Measurement*: The *Three-Component Model of Measurement* states that the measurement and the modelling of all *Scientific Systems* must consider three separate *Components* and their *Physical States* established in the *Context* of the Study: the *Observer*, the *Observed*, and the *Standard Metrics*. To illustrate, the *Observed* and the *Observer* and their related *Properties* correspond to *Relativity* while the *Standardized Metrics* correspond to that of the *Center of Mass Reference Frame* of *Newtonian Mechanics*. How these different *Components* inter-relate are shown in Strobel [3]. *Context* has an extended expression for the *Physical Properties* of a *Physical System* in that the *Experimental Strategy* is critical to the resulting *Observations* and the *Mathematical Representation* of the *Physical Properties* of the *Physical System*.

- *Quantum Fundamental*: All *Physical Properties* are *Mathematically Represented* as the *Multiplication* by a *Numeric Value* to a *Standardized Quantum* for the corresponding *Physical Property*. For example, a k meter distance is defined as $k \times 1m$ where $1m$ is the *Value* of the *Quantum*. This *Value* is a *Standardized* quantity and in this example it is *Normalized*. The *Quantum* for a *Standardized Metrics* is established by the *Context* of the *Scientific Study*.

- *principium universala measuram*: There exists a set of *Standardized Metrics* by which the *Physical Properties* of all *Scientific Systems* are *Measured* and *Represented Mathematically*. Specific examples for such *Scientific Studies* are those using standardized measurements of *Length* (1 *meter*), *Time* (1 *sec*), and *Mass* (1 *gram*). Different *Scientific Studies* may use different *Standardized Metrics*. These are *Mathematical Constructs* and can have *Normalized Values* arbitrarily chosen to place them in a suitable *Mathematical Framework*.

- *formulationum constantes sunt constantes*: There exists a set of *Universal Calibration Constants* by which the *Physical Properties* of all *Scientific Systems* can be *Calibrated* from *Mathematical Representations* to *Real World Physical Dimensions* and then back as necessary. All *Scientific Studies* uses these *Real World Physical Calibration Constants* to cast the *Physical Problem* into a *Mathematical Problem* and conversely.

... [*Statement 1*]

The full set of these *Most Fundamental Principles* establish all components of a *Scientific Study* and the *Scientific Study* can trace it's *Philosophical* roots to every one of these *Principles*. They are distinguished to be *Most Fundamental* because there are no other *Principles* from which these *Principles* can be derived *Ipso Facto*, excluding that by an *Inverted* line of *Reasoning*. Thus, the *Most Fundamental Principles* are accepted *Defacto–Apriori* while those otherwise will be consequent of other *Principles*, and therefore accepted *Ipso Facto*.

Strobel determines several *Direct Consequence Principles* that can be reasoned directly from *Principles Most Fundamental Ipso Facto*.[2, 3, 10] The following *Direct Consequence Principles* are identified below as necessary for this discussion:

- *Context*: The *Context* of a *Scientific Study* is established by features of the *Subject* and features of the *Scientific Experiment*, including—how the *Measurements* are performed, how the *Model* is *Mathematically Represented*, the *Standardized Metrics* used, and etc.

- *critica contextus*: The *Physical Properties* of all *Scientific Systems* are based on the *Context* of the *Scientific Study* and governed by the *Mathematical Properties* of these *Contexts*.

- The *Law of Context*: The *Physical Laws* governing the *Properties* of a *Physical System* as derived in any *Scientific Study* is established by the *Transformation* between differing *Contexts* that are part of that *Study*.[2]

... [*Statement 2*]

Strobel goes on to present a *Direct Consequence Principle* referred to as the *Fundamental Equation* (*FEq*) paraphrased below:[10]

> The *Fundamental Equation* (*FEq*): As a *Direct Consequence Principle* of *Most Fundamental Principles*, the *Physical Properties* of each and every *Physical System* of

the *Universe* is governed by the *Mathematical Relationship*, the *FEq*:

$$m\tau M, \tag{4}$$

where m defines the *Measurement Process* and M defines the *Model Space Representation*—both these components of a *Scientific Study* are *Represented Mathematically* on *Metric Spaces*. τ is some *Transformation* that defines the *Mapping* between the two *Metric Spaces* and by this *Direct Consequence Principle*, define the *Laws of Physics* governing each *Physical System*. *Equation* (4) has an *Inverse Operation*:

$$M\tau^{-1}m. \tag{5}$$

It is acceptable for *Equation* (4) to be identified as the *Forward Representation* and *Equation* (5) the *Inverse Representation*. It is equally acceptable for the reverse to be considered the standard for any given *Scientific Study* should that serve the best interests of the investigation.

... [*Statement 3*]

In the *Real World*, there will always be an *Inverse* and a *Forward Mathematical Representation* that is *Finite Everywhere and Always*, and can be *Mathematically Represented* to be *Finite* and *Non Zero* in the *Infinite Limits*, to every *Measureable Law of Physics*.

Geometric **Treatment of** *Newton Mechanics* **and** *Quantum Mechanics*

By the *Law of Context*, the *Physical Properties* of a *Scientific System* are established by the *Context* of the *Scientific Study*, and the *Context* is established by *Mathematical Representation* based on a *Geometric Structure* established by the *Three Component Model of Measurement*. The *FEq* states that the *Mathematical Transforms* between *Contexts* establishes the *Physical Properties* for all *Scientific Studies*. Thus, all *Laws of Physics* governing the *Physical Properties* of all *Physical Systems* are *Represented Mathematically* based on the *Context* of the *Study*. By *Quantum Fundamental*, all *Physical Properties* are some *Multiple* of *Standardized Metrics* that may be *Calibrated* or *Normalized* and thus the *Physical Laws* governing the *Physical Properties* of a *Scientific System* are the *Multiples* of the *Transforms* of these *Calibrated Standardized Metrics*. Since these *Standardized Metrics* are *Universal* to the *Context* of the *Scientific Study* and define the *Physical Properties* of the *Observables* they describe, they are the *Universal Laws* of *Physics* for that *Context*.

A *Scientific Study* may have two *Model Space Representations* resulting in two governing sets of *Physical Laws*. If both *Scientific Studies* follow a process in which the *Measurements* are executed in a common *Context*, then there exists a *Transformation Mapping* one *Mathematical Representation* to the other thereby representing the *Physical Properties* of the *Physical System* each *Model Spaces*. This *Transformation* establishes the *Laws of Physics* governing this particular *Scientific System* as they relate to alternative *Mathematical Contexts*. This applies to those *Laws of Physics* that may appear to be incompatible.

Consider a *Scientific Study* using both *Quantum Mechanics* and *Newtonian Mechanics Representations* for the same *Physical System*. Writing the statement of the *FEq* for both *Mathematical Representations*:

$$m_Q \tau_Q M_Q,$$

and

$$m_N \tau_N M_N.$$

If the *Physical Properties* of both are measured in the *Center of Mass Context* which is the *Standardized Newtonian Mechanics Context*, they must yield the same result, and thus:

$$m_Q = m_N,$$

$$\therefore M_Q = \tau_Q^{-1}(\tau_N M_N), \tag{6}$$

and

$$\tau_{QN} = \tau_Q^{-1}(\tau_N), \tag{7}$$

with the *Inverse* defined as:

$$\tau_{NQ} = \tau_{QN}^{-1} = \tau_N^{-1}(\tau_Q). \tag{8}$$

Establishing the Mathematical Context

The *Transformation* of *Equation* (7) has some *Forward* and some *Inverse* ($\tau_{NQ} \equiv \tau_{QN}^{-1}$) form that is *Finite* and exists *Always, and Everywhere* and since the *Three-Component Model of Measurement* establishes the *Mathematical Representation* to be *Geometric* in nature, then the resultant derivation will at least have a *Geometric Representation.* To determine this *Transform* it is first necessary to establish the *Context* of the *Scientific Study.* The most important components will be the *Metric Spaces* with the *Standard Metrics* that *Mathematically Represent* the *Physical Properties* of the *Elements* in that *Context.* There will be a number of these *Metrics* but they can be categorized into the *Primary Standard Metrics*, the *Principle Standard Metrics*, the *Reference Standard Metric*, and the related *Composite Metrics.*[1] Of particular interest is the *Principle Standard Metric* which defines the *Principle Physical Property* from which the *Physical Properties* of interest to the analysis will be derived. In the *Newtonian Mechanics Reality*, the *Principle Standard Metrics* is the *Generalized Position Metric* which includes the *Position* of the *Elements* of the *Study* and all N^{th} *Order Differential Forms* of those *Position Metrics.* Every *Component Order* is defined in it's own *Normalized Three Dimensional Euclidean Space* with one vertical and two *Horizontal Dimensions* defined by a *Unit Valued Orthogonal Space Basis Metric.*[1]

The *Space Basis Metrics* are *Standardized Metrics* that define *States* of *Position* of the *Observer*, the *Observed*, and the *Standards.* This must be *Generalized* to all *Differential Orders* N including *Velocity*, *Acceleration*, and etc., up until the *Infinite Order* where the N^{th} *Component* is defined as $M \in [1-3]$ of the *Three Dimensional Euclidean Space* for the N^{th} *Differential Order* $N \in [1, \infty]$. Each of these *Sub Space Basis Metrics* are *Components* M in *Order* where *Component* N are *Parrallel* with every other *Sub Space Basis Metrics Component* M in *Order* R, $R \in [0, \infty]$. *Mathematically*, this *Metric Space* can be defined as:

$$M_{NS_b} = \{M\} \mid M \equiv \{\{x_j\}_i \mid j \in [1,3]\&$$

$$i \in [0,\infty]\&(x_{s/(j,i)} \parallel x_{s-(j,k)}\&x_{s/(j,i)} \perp x_{s/(l,i)} \forall l \neq j \forall k \in [0,\infty])\}, \tag{9}$$

and is identified to be an *Infinite Set of Three Dimensional Euclidean Spaces.* The *Zeroth Order Component* is the *Position State.* Any deviation from this *Normalized* and *Orthogonal* arrangement can be *Transformed* into a *Metric Space* that is *Normalized* and *Orthogonal* and then treated equivalently making the original *Metric Space* a valid *Universal Representation* for all such *Contexts.* There is no loss of *Generality* in doing this.

The *Quantum Mechanical Context* must likewise be established. The *Mathematically Equivalent Metric Space* to the *Quantum Mechanical Metric Space* is *Calibrated* and is the *Infinite Dimensional Euclidean Space* with each *Space Basis Metric Purpendicular* to every other *Space Basis Metric.* The *Space Basis Vectors* are the *Generalized Momentum Metric* and the *Metric Space* is defined as:

$$M_{QS_b} \equiv \{|\overset{i}{\vec{\rho}_s}| \mid |\overset{i}{\vec{\rho}_s}| \equiv |\overset{i}{\vec{x}_s}| \cdot m \in \mathcal{E}_\infty \mid |\overset{i}{\vec{x}_s}| \perp |\overset{j}{\vec{x}_s}| \forall j, i \in [0,\infty]\&i \neq j\}. \tag{10}$$

It is an *Infinite Dimensional Euclidean Space* with each N^{th} *Order Space Basis Metric* the *Calibrated* N^{th} *Order Generalized Differential Order* of the *Momentum State.* The *Calibration* factors

[1] *Principles* of *Relativity* ignored with due respect.

for the *Space Basis Vectors* are the *Universal Calibration Constant* for *Mass*, the *Universal Calibration Constant* for *Distance*, and the *Universal Calibration Constant* for *Time*. *Time* is the *Reference Calibration Constant* by convention and is arbitrarily given the *Value* of 1 with no loss of *Generality.*[1] Figure (1) illustrates the *Metric Spaces* used to establish the *Context* for this problem. The *Metric Spaces* involved are the *Infinite Set of Three Dimensional Euclidean Spaces* that are *Normalized* and the *Ordered Infinite Dimensional Euclidean Space*, which is *Calibrated* with h.

The *Metric Spaces* defining the *Measurement Space Representation* and the *Model Space Representation* are both part of the *Experimental Design.* If these components of the *Scientific Investigation* are changed, the *Experiment* will be different and consequently the *Experimental Results* and the *Laws of Physics* establishing the *Physical Properties* of the *Physical System* will be different in appearance.

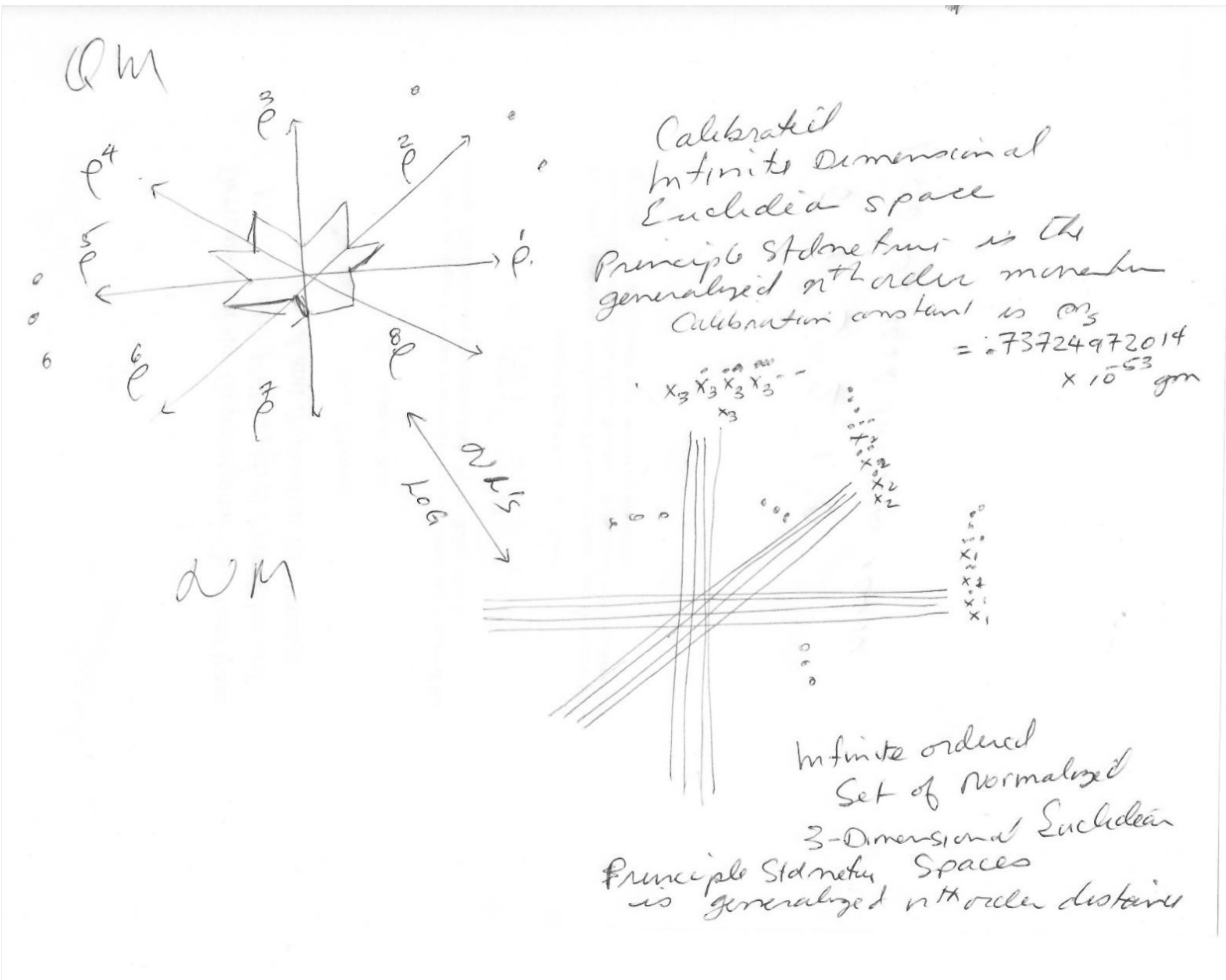

Figure 1: The *Metric Spaces* defining the *Context* in the Derivation.

At this point, the following conclusions can be drawn:

- The *Mathematical Formulations* for the *Laws of Physics* in *Quantum Mechanics* and *Newtonian Mechanics*, based upon the *Normalized Metric Space* M_{NS_b} and the *Calibrated Metric Space* M_{QS_b} are the *Transformations* between the *Calibrated Ordered Infinite Dimensional Euclidean Space* based on the *Generalized Momentum Space Basis Metrics* and the *Metric Space* comprised of an *Infinite Set of Three Dimensional Euclidean Spaces*, based on the *Normalized* N^{th} *Order Differential* of the *Position Metrics*.

- As this is a *Transformation* between *Generalized Momentum* and *Generalized Position*, the *Units* of the *Transform* will be the *Standardized Value* for *Mass* for the *Forward Transform*, thereby *Calibrating* the *Quantum Mechanical Metric Space* and the *Multiplicative Inverse* for the *Standardized Value* for *Mass* for the *Inverse Transform*, thereby *Normalizing* the *Newtonian Metric Space*. In this way the *Principle Standard Metrics* map between each other. This *Mathematical Relationship* defines the *Direction* of the *Transforms*.

At this point, there is no other consideration as to the *Context* to which this applies other than the two *Metric Spaces* and the *Principle Standard Metrics* involved, thus, this *Context* is *Independent* of additional *Constraints* and is *Universal* to all *Physical Systems* defined in these terms.

It is necessary to identify the *Standardized Metric* under the *Transform* τ_{QN}. The first common feature for both *Metric Spaces* are the 0's that define the *Origin* for the 0^{th} *Order Differential* of *Position* in the M_{NS_b} *Metric Space* and the 0^{th} *Order Metric* $m \cdot |\overset{0}{\vec{x}}(0)|$ for the M_{QS_b} *Metric Space*. This defines the *Special Newton Mechanics Context*. This is true since a stationary *Observable* in the *Newtonian Mechanics Context* has *Zero* momentum in the N^{th} *Order*, which defines the *Zero Momentum State*. Thus, they are *Directly Equivalent* at this value:

$$|\overset{0}{\vec{\rho}_s}(0)| = m \cdot |\overset{0}{\vec{x}_s}(0)| = 0. \tag{11}$$

A second common feature will be *Unit Valued* for the *Space Metric* d in the *Normalized Metric Space* and in particular the *Normalized Distance from the Origin* which will *Map Equivalently* from the common *Origins* between *Metric Spaces* to the *Unit Value* calibrated to the *Calibrated Value* in the *Quantum Mechanics Context*. This maps a *N-Dimensional Unit Sphere* to a *N-Dimensional Calibrated Sphere* based on the *Standardized Metric* the *Distance from the Origin* between both *Metric Spaces* transform as:

$$|\overset{0}{\vec{d}_\rho}(s - (h - 0))|\tau(m \cdot |\overset{0}{\vec{d}_x}(1 - 0)|). \tag{12}$$

The *Space Metric* is defined by the *Pythagerous Theorem* and thus the *Transform* for the *Distance from the Origin* from *Equations* (11) and (12). For the *Infinite Set of Three Dimensional Euclidean Space* the values are:

$$|d_{NMn}|^2 = \sum_{i=1}^{n} \sum_{k=1}^{3} \sqrt{x(1-0)^2},$$

$$|d_{NMn}|^2 = \sum_{i=1}^{n} \sqrt{3}. \tag{13}$$

For the *Infinite Dimensional Euclidean Space*:

$$|d_{QMn}|^2 = \sum_{i=1}^{n} n^2. \tag{14}$$

The *Identity* is constructed as the N^{th} *Order Term* of a *Partial Sum* from the *Euclidean Distance* (*Forward* and *Inverse*), taking it to the *Infinite Limit* and exploiting a number of properties in the *Limit* and with *Constraints* related to the *Partial Sums* and initial *Infinite Series*. The *Mathematical Properties* making the *Transform* possible are that the *Infinite Limits* must be expressible as *Finite Valued* although not necessarily under every *Transform*. This is generally the case if the *Series* from which the *Partial Sums* are constructed are *Countable*. [6, 9]

There are as many ways of deriving the *Transform* τ_{QN} as there are ways of expressing the *Pythagerous Theorem*. A total of twelve strategies for these derivations are due to Strobel.[ibid] Most explicitly depend on the *Euclidean Distance*—the *Space Metric* of the *Metric Spaces*—and all depend on the *Infinite Limits* of the *Standard Metrics* for *Generalized States* for the *Elements* in the *Analysis*.

These derivations are independent of the system of units. For measurements in independent *Scientific Studies* using SI and FPS units, they compare favourably when considered after converting to common units of *Measure*. The result shows a consistency in the calculated value G^d to 2 parts in 10^6 which is significantly more consistent than results from the comparison of multiple direct measurements of G^c.

Strobel [9] develops and outlines the following *Mathematical Structures* used for the derivations:

- An adapted *Sierpinski's Triangle* collapsing a *Infinite Dimensional Standard Metric* from an *Infinite Set of Three Dimensional Spaces* to a *Vector* in a *Infinite Dimensional Euclidean Space*,
- A *Series* constructed from *Circumscribed Triangles* creating a *Fractal Series* then evaluated in the *Infinite Limit*,
- From the *Normalized Value* of 1 which leads directly to *Pythagerous Theorem*, leading to *Ptolemy's Theorem* and from there to the result,
- From the *Lucas Numbers* (*Inverse*), and the *Fibonacchi Numbers* (*Forward*),
- From the *Harmonic Series* ,
- From the *Euler-Zeta Function*,
- From *Finite Differences* and the *Calculus Of Differences*,
- From *Pascal's Triangle*,
- From a *Philosophical* development from Strobel [2, 3, 6] referred to here. This development is important as it establishes these as *Newton's Laws*, the *Law of Gravity*, and the *Gravitational Constant* G^c and that the underlying properties are as presented.

These developments are similar since many share a number of common steps and it is possible to define new variations simply by changing the starting point in an existing strategy. The adapted *Sierpinski's Triangle* for example, is one way to extract *Pythagerous Theorem*. The result is an *Infinite Dimensional Matrix* that is *Calibrated* into *Infinite Dimensional Vectors*.

The *Mathematical* analog to the *Physical Law of Gravity* is:

$$\{\overset{l}{1} \overset{\tau}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot \left(\frac{\overset{0}{N}}{l!} \right) . \tag{15}$$

In *Newtonian Mechanics*, but with the language of *Quantum Mechanics*, the representative state *Metrics* in the *Newtonian Mechanics* representation given in terms of the *Quantum Mechanics Representation* is:

$$|\overset{l}{1}|^4 = \left(\frac{h}{m \cdot c^2} \right)^4 = \left(\frac{G}{N} \cdot \frac{1}{m} \right)^4 , \tag{16}$$

has *Normalized Value* $(1)^4$, with the implicit statement for emperical h. This is a *Statement* that *Newton's Laws* and the *Law of Gravity* are complete representation for the *Laws of Physics* without any knowledge of those of *Quantum Mechanics* since h is built into G^d. It also shows that there is a *Quantum Mechanics Mathematical Representation* in *Newtonian Mechanics* and conversely.

The analogous *Mathematical* construct to the *Law of Gravity* and *Newton's Laws*, in a purely *Mathematical Analysis* discussed so far, is a *Vector* of *Normalizing Values* mapping *Unit Spheres* about the *Origins* of two *Infinite Dimensional Metric Spaces.* The *Mathematical Identity* defined by the *Standardized Metric* is the *Distance from the Origin Metric*, defined by the *Euclidean Distance* that is the *Space Metric* for these *Metric Spaces.* The *Distance from the Origin* in the *Ordered Normalized Infinite Dimensional Euclidean Space* is *Mapped* onto the *Distance from the Origin Metric* in an *Normalized Ordered Infinite Set of Three Dimensional Euclidean Spaces* using the *Value* $m \equiv 1$. The *Normalized Metric* is:

$$\{\overset{l}{N} \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot \left(\frac{\overset{0}{N}}{1^{(l)} l!} \right), \tag{17}$$

where $[0 \le l \le \infty]$ and then

$$\{\overset{l}{1} \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot \left(\frac{\overset{0}{N}}{\overset{l}{1} l!} \right). \tag{18}$$

The resulting *Transformation* in *Normalized Space* is:

$$|\overset{1}{N}| = \frac{4!}{2!} \cdot e^{\frac{1}{4}\left[\ln\left(\frac{1}{\varphi} \cdot \frac{1}{2 \cdot e^4}\right) - 1\right]}, \tag{19}$$

which reduces to:

$$|\overset{1}{N}|^4 = \left[\frac{1296}{(1+\sqrt{5}) \cdot e^5} \right]^4 = (1.2816774372....)^4, \tag{20}$$

where the $\overset{1}{N}$ is a scalar factor applied to the *Vector* in *Equation* (18).

The *Value* for $|N|$ in *Equation* (20) is from a *Vector Formulation* where the *Components* establish the *Constants* for the *Transformation* in each *Differential Order*, *Component* by *Component* and where both *Metric Spaces* are *Spanned* with *Normalized Space Basis Vectors.* This is the *Mathematical Equivalent* to G^d.

The *Domain* and *Range* of these *Maps* are the *Unit Spheres* about the *Origin* in the *Normalized Metric Spaces.* These *Unit Spheres* are *General* K^{th}*–Dimensional Objects* defined by the *Differential Order* $k|k \in [0, \infty]$. They are defined at the *Unit Value* of the axis's but when the full *Unit Sphere* is considered for this *Transform*, the *Complex Numbers* become useful in their *Representation.*

The *Inverse* to *Equation* (19) is:

$$|\overset{1}{1}| = \varphi \cdot e^{4\left[\ln\left(\cdot|\overset{2}{N}|/\frac{4!}{2!}\right)\right]+1}. \tag{21}$$

When a *Calibrated Metric Space* is considered, the *Space Basis Metrics* must be *Calibrated* with *Real World Calibration Constants.* Strobel [1] discusses the different types of *Universal Constants* and identifies the three *Primary Calibration Constants* as *Time*, *Distance*, and *Mass. Time* is the *Reference Universal Constant* and is arbitrarily given the *Value* One. There are two other *Universal Calibration Constants* of interest to this discussion, that being for the *Generalized*

Momentum Metric (from h) and the *Generalized Distance* (from the *Speed of Light-c*). Strobel shows that *Einstein's Mass—Energy Equation* can be reduced to:

$$\frac{m}{t_s} = h \cdot \left(\frac{1}{c \cdot t_s}\right)^2 = 0.73724972014... \times 10^{-53} gm/sec. \tag{22}$$

This defines the *Universal Calibration Constant* for *Mass*, analogous to the *Speed of Light-c* for *Generalized Distance*. *Einstein's Mass—Energy Equation* is written explicitly as:

$$h = \frac{m}{t_s} \cdot \left(\frac{d_s}{t_s}\right)^2, \tag{23}$$

where $d_s \equiv c \cdot t_s$ defines the *Primary Universal Constants* with respect to the *Reference Standard Constant*—and similarly for the *Universal Calibration Constant* for *Mass*.

The *Universal Calibration Constant* for *Mass* is used to *Calibrate* the *Transform* of *Equation* (19) can be written as:

$$|\overset{1}{G}| = \left(\frac{4!}{2!}\right) \cdot e^{\frac{1}{4}\left[\ln\left(\frac{m}{\varphi} \cdot \frac{1}{2 \cdot e^4}\right) - 1\right]}. \tag{24}$$

The *Inverse* to *Equation* (24) is:

$$|\overset{1}{h}| = c^2 \cdot \varphi \cdot e^{4\left[\ln\left(\frac{|\overset{1}{G}|}{\frac{4!}{2!}}\right)\right] + 1}. \tag{25}$$

Equation (24) can be reduced:

$$|\overset{1}{G}|^4 = \left[\frac{1}{2} \cdot \left(\frac{4!}{2!}\right)^4 \cdot \frac{h}{\varphi \cdot e^5 \cdot c^2}\right]^4, \tag{26}$$

which can be reduced to:

$$|\overset{1}{G}|^4 = \left[\frac{1296}{(1+\sqrt{5}) \cdot e^5} \cdot m\right]^4 = (0.667855325064921... \cdot 10^{-13} gm/sec)^4. \tag{27}$$

The *Transforms* are:

$$\{\overset{l}{G} \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot \left(\frac{m \cdot \overset{0}{G}}{r_s^{(l)} l!}\right), \tag{28}$$

where $[0 \leq l \leq \infty]$ and with $M \equiv \sum^i {}^i m$ and then:

$$\{m \cdot \overset{l}{r_s} \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot M \cdot \left(\frac{m \cdot \overset{0}{G}}{\overset{l}{r_s} l!}\right), \tag{29}$$

while the *Calibrated Standardized Metric* is:

$$\overset{k}{G} = (-1)^{(k+1)} \cdot \frac{k}{2} \cdot \overset{0}{G}. \tag{30}$$

Equation (25) will recover the same h since the experimental value for h is used to generate the *Forward Calculation* of G^d in *Equation* (24). In *Newtonian Mechanics*, but with the language of *Quantum Mechanics*, the representative *State Metrics* in the *Newtonian Mechanics* representation corresponding to terminology of the *Quantum Mechanics Representation* is:

$$(|\overset{l}{1}|)^4 = \left(\frac{h}{m \cdot c^2}\right)^4 = \left(\frac{G}{N \cdot m}\right)^4, \tag{31}$$

has *Normalized Value* $(1)^4$, with implicit statement of h and in all units of measure.

From these results it can be deduced that the *Law of Gravity* is the *Inverse Operation* to *Newton's Laws* once both are expanded to all *Differential Orders*. Since the terms are multiplied by a factor of $0.73724972014 \cdot 10^{-53} gm/sec$ and divided by a factor of $c^k \cdot k!$ terms of the 3^{nd} *Differential Order* can be ignored in the *Newtonian Context*:

$$\left\{ m \times \overset{l}{r_s} \overset{\tau}{\leftrightarrow} 1 : l \in [0,2] \right\} = (-1)^{(l)} \cdot M \cdot \left(\frac{m \cdot \overset{0}{G}}{\overset{l}{r_s} \cdot l!} \right), \tag{32}$$

where the terms drop off as:

$$O^3 \left(\frac{0.54784235257... \times 10^{-83}}{(0.299792458... \times 10^9)^l \cdot (l+2)!} \right) l \in [1, \infty]. \tag{33}$$

Equations (18) and (24) are *Normalized* and thus hold for all *Scientific Studies* performed using only *Normalized Metrics*. *Equations* (24) and (25) are *Calibrated* in the *Ordered Infinite Dimensional Euclidean Space* but *Normalized* in the *Ordered Normalized Infinite Set of Three Dimensional Euclidean Spaces* and thus hold for all *Scientific Studies* performed in this *Context*. The development here makes no assumptions on the *Calibration Factors* and thus holds for all *Calibrations*. This defines *Equations* (24) and (25) to be *Modular Forms* and thus are *L–Function* representations for *Infinite Series* of *Partial Sums*.

The calculations above have assumed SI units, however, there is no reason FPS Units cannot be used directly. The calculations are independent of the system of units and independent *Scientific Studies* using SI and FPS units are compare after converting to common units of *Measure*. From *Equation* (24) using $h = .15723916... \times 10^{-32}\ ft \cdot lb/sec$ and $c = .983571036... \times 10^8\ ft/sec$:

$$m_{sFPS} = \frac{h_{FPS}}{c^2_{FPS}} = 1.6263588211... \times 10^{-50} lb/sec \tag{34}$$

If this number is multiplied by the conversion factor between *Mass* in SI units and FPS units ($1lb = 0.453592...gm$) and used in *Equations* (20) and (24):

$$G'_{FPS} = 0.66785546992... \times 10^{-13} gm/sec. \tag{35}$$

The values of G^d calculated in *Equations* (24) and (35) from two independent measurements of h gives an indication of the accuracy for the calculated value for G^d. The difference is:

$$|0.66785546992...10^{-13}| - |0.66785532506... \times 10^{-13}| = 0.11... \times 10^{-4}\%, \tag{36}$$

which is roughly a variance of approximately 2 parts in one million.

Two independent values for G^c in two separate experiments are reported with values of $0.6674184... \times 10^{-13}$ and $0.6674484... \times 10^{-13}$. [12] These results show variations in G^c in the fifth decimal position. The result here from using *Planck's Constant* shows a significantly improved consistency in the calculated value of G^d. *Equation* (36) suggests a significantly more accurate value for G^d as:

$$\approx 0.667855397 \times 10^{-13} + / - 0.00000011 \times 10^{-13} gm/sec. \tag{37}$$

A proper investigation of this is necessary to produce a more rigorous evaluation for the accuracy. When this accuracy is compared with that of two independent measurements of G^d,[IBID] in this case by the same researchers and reported in the same publications[12], their results are not within experimental error of each other. This contradiction results from the change in *Context* as the *Physical Properties* of the *Elements* establishing the *Context* evolve in time, particularly their positions in terms of the *Context* of the *Earth's* orbit with respect to the *Sun* and *Moon*. A rough calculation for the variation possible for measured values of G^c over the *Orbit* of the *Earth* while considering the *Moon* 's influence, gives an approximate value of 0.7%.

The result of *Equation* (35) shows that this development produces a *Universal Constant* that is transformed from *Context* to *Context* by the conversion factor for the *Primary Calibration Constants* for the two *Transform Metrics.* Strobel [7] shows that this *Mathematical Representation* for *Gravity* and *Newton's Laws* is a *L–Function* representation of a *Modular Form.*

It is shown that the *Law of Gravity* is a *Mathematical Identity* that *Maps* the *Distance from the Origin Metric* defined by the *Space Metric* of the *Ordered Infinite Dimensional Euclidean Space* that is *Calibrated* by the *Primary Calibration Constant* for *Mass*, into the *Distance from the Origin Metric* in the *Normalized Ordered Infinite Set of Three Dimensional Euclidean Spaces.* The *Law of Gravity* is the *Inverse* to *Newton's Laws* which perform the *Transformation* in the opposite direction. These *Transforms* have always been associated with the *Physical Properties* of *Planets*, *Stars*, and *Galaxies*, but they generally apply to any *Physical Systems*, including *Quantum Mechanical Contexts.*

The currently understood G^c is *Context Dependent* as it is treated *Conventionally.* Any *Scientific Study* performed with differences in the *Geometric Configuration* that changes the *Context* will result in a different measured *Value* for G^c and most any other *Physical Property* for that matter. If G^c is accepted as a invariable *Universal Constant*, then measured over time or space, it will appear to violate this property as the *Context* changes. There will always exist in a specific case for every *Scientific Study* for which there will be a *Universal Constant* G_{SM} of the "*Special Newtonian Context.*" This is when the *Observer*, the *Observed*, and the *Standards* in terms of the *Three Component Model of Measurement*, are *Coincident* in *Time* and *Space.* h if it is defined as the *Universal Constant* established in the *Special Newton Mechanics Context* in the *Context* of all *Scientific Investigations*, is a *Universal Calibration Constant* by this definition. Every *Quantum Mechanics Scientific Investigation* will be based on the same *Universal Calibration Constant* for h since they are performed in the *Center of System Context.*

Expressing *Physical Properties* in terms of the *Standardized Metrics*

In any given *Scientific Investigation*, the *Physical Properties* of the *Elements* are stated in terms of the *Standardized Metrics* and the *Universal Physical Constants* in the case for *Calibrated* results. The *Principle Standard Metrics* provide the *Standardized Metric* and the *Physical Properties* are expressed as a *Multiple* of the *Standard Metric.* For example, in the *Newtonian Mechanics Context*, the *State of Position* ($\overset{0}{x}$) of an *Observable* is expressed in terms of the *Standardized Position State Metric* ($\overset{0}{x}_s$):

$$\overset{0}{x}_{nm} = |\overset{0}{x}_{nm}| \cdot \overset{0}{x}_{nm_s}. \tag{38}$$

For the *Newtonian Mechanics Context*, this value is *Normalized* ($\equiv 1$). For the *Quantum Mechanics Context*, the *State of Position* is expressed in the *Standardized Position State Metric* which is *Calibrated* by the *Speed of Light-c.*

$$\overset{0}{x}_{qm} = |\overset{0}{x}_{qm}| \cdot \overset{0}{x}_{qm_s}. \tag{39}$$

The *Standardized Metrics* provide the measurement standards for the *Scientific Investigation* and are part of the *Context.* This is the *Fundamental Principle* identified as *Quantum Fundamental.*

To express a *Quantum Mechanics Position* state in a *Newtonian Mechanics Context*, the *Transformed State Metric* is used:

$$\overset{0}{x}_{nm} = |\overset{0}{x}_{qm}| \cdot \tau(\overset{0}{x}_{qm_s}). \tag{40}$$

The *Transform* is a *Mapping* of the *Space Basis Metrics* and any *Physical Property* for an *Observed* is the multiple of the *Transform Space Basis Metric* by *Quantum Fundamental.* This is a loose definition for *Modular Forms* and a rigorous proof this property is given in Strobel[7].

Metric	*Newtonian Value* (*Defacto–Apriori*)	*Calibrated Value* Calculated	*Measured*
Distance	$1m$	*Emperical* (m/sec)	$0.299792... \times 10^{10}(m/sec)$
Mass	$1gm$	*Emperical* (gm/sec)	$0.737249... \times 10^{-53}(gm/sec)$
Time	$1sec$	$1(sec/sec)$	*Defacto–Apriori*
Momentum†	$1gm \cdot m/sec$	*Emperical* $(gm \cdot m/sec)$	$0.662607...^{-36}(gm \cdot m/sec)$
G^c	$1.28167...(Normalized)^{\ddagger}$	$0.667855397... \times 10^{-13}(gm/sec)$	$0.66743... \times 10^{-13}(gm/sec)$

Table 2: The *Calibration Values* for the *Standardized Metrics*
This table contains the calculated values of the *Calibrated Standard Metrics* which are the *Transformed Unit Values* from the *Newtonian Mechanics Context* to the *Quantum Mechanics Context*. All values are with respect to *Unit Time* (the *Reference Standard Metric*).
†—since the calibrated value for *Mass* is calculated from the measured value for *Planck's Constant*, the calculated value for the calibrated *Planck's Constant* will be the same as the measured. The *Normalized* value for *Momentum* is 1.
‡The *Newtonian Value* for G^d is taken to be the *Transformation* (N) between *Normalized Metric Spaces* to be consistent in principle with the treatment of *Normalized Values* as *Standardized Magnitudes* in the *Newtonian Context.*

There is an equivalent calculation of certain *Standardized Metrics* as that expressed in *Equation* (38). In the cases where the measured values are used to calculate other *Standardized Metrics*, there won't be a calculated value. The results are tabulated in Table (2).

Treatment of the *Masses* of *Planets*

The concept of *Context* serves to reconcile apparently conflicting theories of *Quantum Mechanics* and *Newtonian Mechanics*, at least based upon the *Fundamental Constants* for each. *Context* can be taken further when considering the conventional calculations for the bulk masses of *Planets.*

Derivations are ultimately *Mathematical* in nature but the *Philosophical Construction* is critical from beginning to end. The *Newtonian Mechanics Context* and the *Quantum Mechanical Contexts* are normally treated differently. *Newtonian Mechanics Contexts* are commonly subject of *Scientific Studies* in different *Contexts* such as the examples for the *Center of Sun Context* and the *Center of Earth Context*. *Quantum Mechanical Contexts* are universally treated in the *Center of System Context*. All are differing *Contexts* from *Philosophical Reasoning* predicting different values for *Context Dependent Universal Constants* such as G^c. Considering this, it is reasoned that the "true" values for the bulk mass of the *Planets* in our *Solar System* are one-half those generally accepted. The results of the comparison are tabulated showing to be consistent with a *Model* of a *Homogeneous* body with a bulk density consistent with the most common density for each *Planet.*[8]

Consider how the *Masses* of *Planets* are computed in *Newtonian Mechanics Contexts.* The calculation of the *Universal Gravitational Constant* from the change of the *Momentum* under acceleration due to the Earth's gravity is measured from values of *Mass* and of force due to gravity where the measurements are made. This is a *Scientific Study* performed in the center of the apparatus that establishes the *Center of System Context* with *Observer*, *Observed*, and *Standards*, represented by the *Experimental* apparatus, in a *Coincident State.* All share the same *Generalized* N^{th} *Order Differential State Mathematically Represented* as a common point in *Time* and *Space*. This is referred to as the "*Special Newton Mechanics Context*".[2, 11] The *Momentum Metric* (ρ) as the *Principle Standard Metric* and is either measured directly or measured in terms of the 1^{st} *Differential Order* of *Generalized Momentum—Momentum*—and the governing equation for this *Principle Standard Metric* is:

$$\{\overset{l}{\rho}_s \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0,\infty]\} = (-1)^{(l)} \cdot \left(\frac{m \cdot \overset{0}{G}_\rho}{\overset{l}{\rho}_s \cdot l!} \right). \tag{41}$$

The *Force* due to *Gravity* in *Equation* (41) for this specific *Context* is divided by $1! = 1$ and the 0^{th} *Order* is divided by $0! = 1$.

Using this measured value for G^c, the *Mass* of any *Planet* is measured based on the *Force* of *Gravity* acting on the *Planet* due to the *Sun*. This *Scientific Study* is performed in the *Center of Sun Context* with the *Generalized Differential Order Form* of *Distance* (x) as the *Principle Standard Metric*. The *State* of the *Observed* is located at the *Position* of the *Planets*. The governing equation is:

$$\{\overset{l}{x}_s \overset{\mathcal{T}}{\leftrightarrow} 1 : l \in [0,\infty]\} = (-1)^{(l)} \cdot \left(\frac{m \cdot \overset{0}{G}_x}{\overset{l}{x}_s \cdot l!} \right). \tag{42}$$

The *Force* due to *Gravity* in *Equation* (42) is constructed from the 2^{nd} *Order Differential Component* of the *Generalized Position Metric* and is divided by $2! = 2$. This is half the value for G^d constructed from the 1^{st} *Order Differential Component* of the *Generalized Momentum Metric* in *Equation* (41)—they will differ by one-half. If it is assumed that G^c is invariant between these two *Scientific Studies*, then the results establishing the *Mass* must account for this by factoring in the difference, i.e.:

$$\left\{ m \cdot \overset{2}{x}_s = \left(\frac{m \cdot \overset{0}{G}_x}{\overset{2}{x}_s \cdot 2!} \right) \right\} ? \left\{ \overset{1}{\rho}_s = \left(\frac{m \cdot \overset{0}{G}_\rho}{\overset{1}{\rho}_s \cdot 1!} \right) \right\}, \tag{43}$$

where the question mark is used to indicate that there is an uncertainty of the *Mathematical Relationship* between the two equations. For *Equation* (43) to be *Directly Equivalent* and for G^c to be *Universally Constant*, i.e., $\overset{2}{G}_x = \overset{1}{G}_\rho$ then:

$$m_a \cdot (C(\overset{1}{G}_x)) = \frac{1}{2} \cdot m \cdot (C(\overset{2}{G}_\rho)),$$

$$\therefore m_a = 2 \cdot m.$$

Therefore, the "apparent mass" (m_a) as determined in the *Center of Sun Context* and the actual *Standardized Mass* (m) as determined in the *Center of System Context* must be related as:

$$m = \frac{1}{2} \cdot m_a. \tag{44}$$

But m is a *Context Independent Universal Constant* and consequently the *Mass Values* in *Equation* (44) must be equal. The only other term in the calculation is G^c which is *Context Dependent* and must account for this factor of two.

The total *Mass* of a *Planet* can be calculated assuming a *Homogenious Bodies* with *Density* constant throughout then calculating the *Bulk Density* from the *Volume*. This is dependent on the *Context* of the *Measurements* for the *Density* which is performed in the *Center of System Context*. To illustrate, if a planet could be put onto a scale as for when the sample for the *Density* was measured, the inconsistancy exposed by *Equation* (44) would be shown directly.

Table (3) shows the calculation based upon the results from the *Gravity* developed here. *Mass* is calculated from the *Density* for the most common material for the individual *Planets*, and compared with the result calculated from the *Center of Sun Context* which is used to establish the conventional *Values* for the *Mass* for the *Planets*. The comparison is shown as a ratio of the two

Planet	*Density*	*Bulk Mass* (m_d) from *Density* †	*Bulk Mass* (m_g) from *Gravity* ‡	*Bulk Mass* Accepted	m_d/m_g	m_a/m_d
Mercury	2.65	1.61×10^{23}	1.64×10^{23}	3.285×10^{23}	0.98	2.04
Venus	2.65	2.49×10^{24}	2.44×10^{24}	4.867×10^{24}	1.02	1.95
Earth	2.75	2.98×10^{24}	2.98×10^{24}	5.972×10^{24}	1.00	2.00
Moon⋆	2.65	5.82×10^{22}	5.87×10^{22}	7.34×10^{22}	0.99	1.26
Mars	2.65	4.48×10^{23}	3.29×10^{23}	6.39×10^{23}	1.36	1.94
Jupiter	0.708	1.01×10^{27}	0.908×10^{27}	1.898×10^{27}	1.11	1.88
Saturn	0.687	5.68×10^{26}	2.65×10^{26}	5.683×10^{26}	2.14	1.00
Uranus	0.708	4.84×10^{25}	4.27×10^{25}	8.681×10^{25}	1.13	1.79
Neptune	0.708	4.46×10^{25}	5.07×10^{25}	10.24×10^{25}	0.88	2.30

Table 3: The *Masses* of the *Planets* from this Analysis (in SI Units-gm)
This table contrasts the *Masses* computed from the bulk *Densities Times Volume* (m_d), the calculations based on *Gravity* from these results (m_g), and the *Masses* accepted in the conventional analysis. The *Densities* used are for the most common material for each *Planet*. The ratio (m_d/m_g) approaches one as the result for m_g approaches m_d.
† The *Bulk Mass* from *Density* is calculated from *Density* × Volume.
‡ The *Bulk Mass* from *Gravity* takes *Context* into consideration.
⋆ The *Moon*'s *Bulk Density* must consider two *Transforms*. One through the *Earth*'s *Orbit* and the other due to the *Moon*'s *Orbit* around the *Earth*.

calculations. As the calculation of *Mass* from *Bulk Density* approaches that from G^d, the ratio approaches 1.

The results show good correlation between the corrected *Masses* for the *Planets* and the *Masses* calculated from the *Density* assuming a *Homogeneious Body*. In the case for the *Moon*, since the calculations are more complex considering the *Orbital* motion of the *Moon* around the *Earth* as opposed to the *Center of Sun Context* directly in establishing the *Context—Center of Earth Context*—the analyses must consider this difference. Strobel provides the properly developed derivation for this case.[8]

The analysis of the outer *Planets* is complicated by their composition of a mix of gases and frozen gases, solid rock, and cosmic dust, and it is *Conjectured* here that their *Densities* are not *Homogeneous*. Their radii are also not clearly defined as they are for the inner *Planets*. *Mars* shows a significant deviation in the result and this is *Conjectured* here due to an interior with a certain amount of gasses and frozen gasses which will lower the *Bulk Density*.

Summary

This discussion summarizes how *Newton's Laws*, the *Law of Gravity* and G^d can be derived from *Geometric* and *Philosophical Arguments* along with the resultant properties from these long accepted *Laws of Physics*. They combine as part of the *Transformation* of the *Distance from the Origin Metric* between the *Normalized Ordered Infinite Set of Three Dimensional Euclidean Spaces* spanned by the *Space Basis Metric* of the N^{th} *Order Differential Form* for the *Generalized Position Metric* of *Newtonian Mechanics* and the *Ordered Infinite Dimensional Calibrated Euclidean Space* spanned by the *Space Basis Metric* of the N^{th} *Order Differential Form* for the *Generalized Momentum Metric* calibrated with h. The latter *Metric Space* is the *Mathematical Representative Metric Space* for the *Quantum Mechanics Context*.

G^d *Calibrates* the *Transformed Metric* in the process between the *Newtonian Mechanics Context* and the *Quantum Mechanics Context*. It is shown to be a *Vector* entity with components given as:

$$\{\overset{l}{G} \overset{\tau}{\leftrightarrow} 1 : l \in [0, \infty]\} = (-1)^{(l)} \cdot \left(\frac{\overset{0}{G}}{l!} \right), \tag{45}$$

where $\overset{l}{G}$ is a *Calibrated Vector* using the *Fundamental Universal Constant* for Mass (0.73724972014...$\times$ $10^{-53} gm/sec$) and the *Geometric Constants Euler's Number* and the *Golden Ratio. Equation* (26) can be written as:

$$(\overset{0}{G})^4 = \left[\frac{1296}{(1+\sqrt{5})\cdot e^5} \cdot \frac{h}{c^2}\right]^4 = \left[\frac{1296}{(1+\sqrt{5})\cdot e^5} \cdot m\right]^4$$
$$= (0.667855325064921... \times 10^{-13} gm/sec)^4. \tag{46}$$

Equation (46) reduces the calculation of G^d to one *Universal Physical Constant* (m), one *Geometric Constant* (*Euler's Number*), and the first two prime *Integers* (1 and 2). There are seven *Mathematical Operations* involved ($\times, -, /, !, +$, power, partial sums). Both G^d and it's *Mathematical* analog N are *Transcendental Numbers* as is *Planck's Constant*. The *Normalized Planck's Constant* is 1.

These results show that G^c and h are not *Fundamental Universal Constants* but can be derived from more *Fundamental Constants*—the *Primary Universal Constants Time*, *Distance*, and *Mass.* They also establish that the *Primary Calibration Constants* for *Mass* and *Time* are as critical to representing the *Physical Reality* as is the *Speed of Light-c*. The *Primary Calibration Constants* for *Time* in all measurement systems is:

$$= 1.000....sec/sec, \tag{47}$$

the *Primary Calibration Constants* for *Mass* in SI units is:

$$= 0.73724972014... \times 10^{-53} g/sec, \tag{48}$$

and in FPS units:

$$= 0.162535741097232... \times 10^{-52} lb/sec. \tag{49}$$

The *Context* establishes that all *Laws of Physics* can be derived from *Geometric* arguments as part of the design of the *Scientific Study*—this by way of the *FEq*.

The *Law of Gravity* is a *Real World* expression of a analogous *Mathematical Construct*. The *Normalized Form* of *Equation* (45) is the *Mathematical Construct* and can be written using the *Normalized Value* for *Mass* as:

$$N^4 = \left[\frac{1296}{(1+\sqrt{5})\cdot e^5}\right]^4$$
$$= (1.2816774372....)^4. \tag{50}$$

Equation (50) is a *Constant* of the *Normalizing Transform* mapping the *Unit Sphere* between the *Normalized Ordered Infinite Set of Three Dimensional Euclidean Spaces* and the *Normalized Ordered Infinite Dimensional Euclidean Space*.

In the *Inverse* operation defined for the *Law of Gravity* is the *Normalizing Factor*:

$$(|\overset{l}{1}|)^4 = \left(\frac{h}{m \cdot c^2}\right)^4 = \left(\frac{G}{N \cdot m}\right)^4. \tag{51}$$

Insight leads to a way of viewing the *Mathematical Relationship* between the *Metric Spaces* of the *Context* of this *Mathemaical System*. Since the *Metric Spaces* are *Spanned* with *Space Basis Metrics* in the *Newtonian Context*, the *Normalized Ordered Infinite Set of Three Dimensional Euclidean Spaces*, the relationships between the *Degrees of Freedom* are covariant in the *Differential*

Orders for the *Parrallel Space Basis Metrics* and *Independent* when they are *Purpendicular*. This is a mixed condition establishing the relationship between the *Covariant* and *Independent Variables* and these are the governing *Newtonian Laws of Physics* for this *Context*. In the *Quantum Mechanics Context*, the *Metric Space* is *Spanned* by an *Infinite Dimensional Calibrated Euclidean Space*, establishing each N^{th} *Order Differential Form* for the *Principle Standard Metric* to be *Linearly Independent* to all other *Orders* as *Mathematically Represented* on each *Axis*. Thus, this *Metric Space Mathematically Represents* each *Degree of Freedom* as being *Linearly Independent* and the *Laws of Physics* establish the *Covariant Properties* of the *Degrees of Freedom* as the *Laws of Physics* for this *Context*.

These ideas here are philosophically at odds with *General Relativity* in that *General Relativity* considers that *Mass* has an action at distance effect as a *Physical Property* in the 2^{nd} *Order Component* of *State*. This particular view point is inherited from *Newtonian Mechanics*. With *Context*, all N^{th} *Order States* of the *Elements* contribute to the *Physical Properties* of all other *Elements* of the *Physical System*, the significance of the amount depending on the *Context*. In the *Quantum Mechanics Context*, an *Infinity* of *Orders* must be considered in general, contrasting with the *Conventional Newtonian Mechanics Context* where only three *Differential Orders* $(0, 1,$ and $2)$ are necessary since higher orders make insignificant contributions at the *Planetary* scale. Strobel shows that on the *Galactic* and *Universal* scale, additional *Differential Orders* become significant.[11]

These derivations alert to the risk in viewing the *Universe* from a *Force Centric* perspective as is *Conventionally* done. A full development of the *Physical Properties* governing the *Physical Properties* of a *Scientific Study* require consideration of all *Differential Orders* in the *Mathematical Analysis*. The *FEq* does not exclude the *Force Centric* viewpoint, it states that as long as the *FEq* is satisfied in any *Real World* application then any resultant *Law of Physics* is acceptable as a *Mathematical Representation* to the *Physical Properties* of *Elements* of the *Scientific Study*. In the case of the *Newtonian Mechanics Context*, terms greater than the force term are insignificant for higher orders. The problem is that *Newton's Laws* and the *Law of Gravity* as it is understood historically, introduces onerous *Constraints* to the *Context* as the *FEq* is concerned, and makes it impossible to reconcile with *Quantum Mechanics* without changing the *Philosophical* underpinnings as was done here.

These results were derived without *Constraints* based on specific applications and thus they can be applied in the *Scientific Study* of all *Physical Systems*. However, the *Metric Spaces* considered here are *Study Specific* and other applications of these ideas must always begin by considering the *Context* of the *Scientific Investigation* including the *Metric Space Representation*. Different *Contexts* can be established using different *Metric Spaces*, *Principle Standardized Metrics*, *Geometric Conigurations* from the *Three Component Model of Measurement*, and by changing the treatment of the *Natural Constraints* and by imposing other *Constraints*.

Although the results presented here lead to a *Mathematical Relationship* between *Newtonian Mechanics* and *Quantum Mechanics* from a *Philosophical Basis* relating the *Universal Calibration Constants* and their *Principle Standard Metrics*, the result does not deal directly with their underlying *Probablistic* verses *Deterministic Philosophical* differences.

The reasoning's here introduce differences for the calculations of the *Planetary* Masses from measurements using G^c. These differences are significant and show that G^c is not a *Universal Constant*, but is *Context Dependent*. Measurements of the bulk mass can be used in a calculation for the mass of a *Planet* and compared with the results for *Gravity* from the ideas introduced here. This calculation supports the position of these ideas yielding values that are within approximately 2% for the calculated masses for the inner planets using values from $Density \times Volume$ calculations.

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